Introduction

This book is about differential equations—a **very** big subject! It is so extensive, in fact, that we could not hope to cover it completely even in a book many times this size. So we will have to be selective. In the first place, we will restrict our attention almost entirely to *equations of evolution*. That is to say, we will be considering quantities q that depend on a "time" variable t, and we will be considering mainly *initial value problems*. This is the problem of predicting the value of such a quantity q at a time t_1 from its value at some (usually earlier) "initial" time t_0, assuming that we know the "law of evolution" of q. The latter will always be a "differential equation" that tells us how to compute the rate at which q is changing from a knowledge of its current value. While we will concentrate mainly on the easier case of an ordinary differential equation (ODE), where the quantity q depends **only** on the time, we will on occasion consider the partial differential equation (PDE) case, where q depends also on other "spatial variables" x as well as the time t and where the partial derivatives of q with respect to these spatial variables can enter into the law determining its rate of change with respect to time.

Our principal goal will be to help you develop a good intuition for equations of evolution and how they can be used to model a large variety of time-dependent processes—in particular those that arise in the study of classical mechanics. To this end we will stress various metaphors that we hope will encourage you to get started thinking creatively about differential equations and their solutions.

But wait! Just who is this "you" we are addressing? Every textbook author has in mind at least a rough image of some prototypical student for whom he is writing, and since the assumed background and abilities of this model student are sure to have an important influence on how the book gets written, it is only fair that we give you some idea of our own preconceptions about you.

We are assuming that, at a mimimum, the usual reader of this book will have completed the equivalent of two years of undergraduate mathematics in a U.S. college or university and, in particular, will have had a solid introduction to linear algebra and to multi-variable (aka “advanced”) calculus. But in all honesty, we have in mind some other hoped-for qualities in our reader, principally that he or she is accustomed to and enjoys seeing mathematics presented conceptually and not as a collection of cookbook methods for solving standard exercises. And finally we hope our readers enjoy working out mathematical details on their own. We will give frequent exercises (usually with liberal hints) that ask the student to fill in some details of a proof or derive a corollary.

A related question is how we expect this book to be used. We would of course be delighted to hear that it has been adopted as the assigned text for many junior and senior level courses in differential equations (and perhaps not surprisingly we would be happy using it ourselves in teaching such a course). But we realize that the book we have written diverges in many ways from the current “standard model” of an ODE text, so it is our real hope and expectation that many students, particularly those of the sort described above, will find it a challenging but helpful source from which to learn about ODEs, either on their own or as a supplement to a more standard assigned text while taking an ODE course.

We should mention here—and explain—a somewhat unusual feature of our exposition. The book consists of two parts that we will refer to as “text” and “appendices”. The text is made up of five chapters that together contain about two-thirds of the material, while the appendices consist of ten shorter mini-chapters. Our aim was to make the text relatively easy reading by relegating the more difficult and technical material to the appendices. A reader should be able to get a quick overview of the subject matter of one or more chapters by just reading the text and ignoring the references to material in the appendices. Later, when ready to go deeper or to check an omitted proof, a reading of the relevant appendices should satisfy the reader's hunger for more detail.

Finally we would like to discuss "visual aids"—that is, the various kinds of diagrams and pictures that make it easier for a student to internalize a complicated mathematical concept upon meeting it for the first time. Both of the authors have been very actively involved with the development of software tools for creating such mathematical visualizations and with investigating techniques for using them to enhance the teaching and learning of mathematics, and paradoxically that has made it difficult for us to choose appropriate figures for our text. Indeed, recent advances in technology, in particular the explosive development of the Internet and in particular of the World Wide Web, have not only made it easy to provide visual material online, but moreover the expressiveness possible using the interactive and animated multimedia tools available in the virtual world of the Internet far surpasses that of the classic static diagrams that have traditionally been used in printed texts. As a result we at first considered omitting diagrams entirely from this text, but in the end we decided on a dual approach. We have used traditional diagrams in the text where we felt that they would be useful, and in addition we have placed a much richer assortment of visual material online to accompany the text. Our publisher, the American Mathematical Society, has agreed to set aside a permanent area on its own website to be devoted to this book, and throughout the text you will find references to this area that we will refer to as the "Web Companion".[1] Here, organized by chapter and section, you will find visualizations that go far beyond anything we could hope to put in the pages of a book—static diagrams, certainly, but in addition Flash animations, Java applets, QuickTime movies, Mathematica, Matlab, Maple Notebooks, other interactive learning aids, and also links to other websites that contain material we believe will help and speed your understanding. And not only does this approach allow us to make much more sophisticated visualizations available, but it also will permit us to add new and improved material as it becomes available.

[1]Its URL is http://www.ams.org/bookpages/stml-51.