▷**Computer Experiment** 1 Using Maple, Mathematica, MatLab, or a standard programming language (BASIC, C, Pascal), write a computer program to solve the pendulum equation numerically, and use it to estimate the period for various initial conditions. Recall that if the length of the pendulum is L then for small displacements from equilibrium, we expect it to behave like a harmonic oscillator whose angular frequency ω is $\sqrt{gL}$. In particular, we expect it to swing back and forth with a period of approximately $2\pi\sqrt{gL}$. How well is the above expectation verified? Does the period depend on the initial displacement?

▷**Computer Experiment** 2 Here are detailed instructions for carrying out the same experiment using 3D-XplorMath.

1) Start 3D-XplorMath, then choose ODE(1D) 2nd Order from the ODE submenu of the Category menu, then choose Harmonic Oscillator from the ODE(1D) 2nd Order menu (the Main menu) that appears to right of the Edit menu.
2) The program will draw a solution for some ODE. How do you find out what the ODE is? Select About This Object from the Action menu. You will see "Xdotdot :=-sqr(aa) $* x - bb * u$", meaning that $\frac{d^2x}{dt^2} = aa^2 x - bb * \frac{dx}{dt}$. Next choose Set Parameters from the Settings menu and you will see that $aa = 1$ and $bb = 0$, so the ODE we are dealing with is $\frac{d^2x}{dt^2} = -x$, i.e., the Harmonic Oscillator with $\omega = 1$, and hence with period 2π. Now close the Formulas window and let's try to verify this period numerically.
3) The ODE Control Panel window should be visible near the bottom of the main window. (If not, choose Yes from the Show ODE Controls submenu of the View menu). In the ODE Control Panel, click the Settings... button. You will see that the initial position is 1, the initial velocity is zero, the Time Span is 20 (meaning that the program will integrate the ODE from the initial time, $t = 0$, until $t = 20$, and that the stepsize is 0.01, meaning that the Runge-Kutta numerical integration algorithm will push the solution forward by 0.01 at each integration step. Change the time span to 6.25, then click Erase in the ODE Control Panel to clear the screen, and then click Create to redraw the solution. (If you like arcade style games, you can leave the Time Span set at 20, and after clicking Erase and Create, click the mouse to stop the integration just before the solution completes a full circle). In any case you can now keep clicking the Step Forward button on the control panel until the current position gets back to the initial position, 1, and the Current Velocity is back to zero. The latter is the more sensitive test, and you should see the sign of the velocity change as the Current Time goes from 6.28 to 6.29. (If you go to far, you can use the Step Backward button to back up.) If you want to find the period more accurately, click on Settings... again, reset the timestep to 0.001 and then keep clicking Step Forward until the velocity changes sign. (The Current Time should read 6.283). Now, click settings again, reset timestep to 0.01 and set the initial position to 0.5, and try to find the period again. You might want to click 2*Scale to double the scale so the radius of the orbit isn't to small—particularly if you use the arcade game approach.
4) Now choose Pendulum from the Main menu. Check that the ODE you are dealing with is in fact $\frac{d^2x}{dt^2} = -\sin(x)$, and then try to get a good estimate for the period when the initial velocity is zero and the initial position is successively $1, 0.5, 0.25, 0.125$. Each time you halve the initial position, double the scale, and note that whereas the direction field is initially quite different from that of the harmonic oscillator, as you successively zoom in it begins to look exactly like the harmonic oscillator direction field.
5) When you have completed this experiment, you should have a good feeling for how to use 3D-XplorMath as a tool for quickly playing with an ODE to discover some of its basic properties.